THERMOELASTIC SHALLOW TRANSFER SHELLS EQUALLY STRESSED OVER THE THICKNESS

M. D. Martynenko, Garib Musa Garib, and S. V. Moroz UDC 539.3:534.1

A condition is derived for membrane deformation of a shallow transfer shell that is in a given force and temperature field and a specific example is examined for its application.

It is known [1] that membrane deformation of shell structures is the most preferable of their operating modes since stresses over the thickness of any section are distributed uniformly. Consequently, the design of such structures in which external force and temperature fields cause no bending stresses is an urgent inverse problem of shell theory. Its solution is given below for shallow transfer shells whose equations are representable in the form

$$x_3 = \varphi(x_1) + \psi(x_2), \quad 0 \leqslant x_1 \leqslant a, \quad 0 \leqslant x_2 \leqslant b, \tag{1}$$

where

$$|\varphi'(x_1)| \ll 1, \quad |\psi'(x_2)| \ll 1. \tag{2}$$

In this case the lines x_1 = const, x_2 = const can be identified with middle surface lines of curvature, where [2]

$$A = \sqrt{1 + [\varphi'(x_1)]^2} \approx 1, \quad B = \sqrt{1 + [\psi'(x_2)]^2} \approx 1,$$
$$\frac{1}{R_1} = -\frac{\varphi''(x_1)}{\{1 + [\varphi'(x_1)]^2\}\sqrt{1 + [\varphi'(x_1)]^2 + [\psi'(x_2)]^2}} \approx -\varphi''(x_1), \tag{3}$$

$$\frac{1}{R_2} = -\frac{\psi''(x_2)}{\{1 + [\psi'(x_2)]^2\}\sqrt{1 + [\varphi'(x_1)]^2 + [\psi'(x_2)]^2}} \approx -\psi''(x_2).$$

If the shell (1) is in the membrane state of stress under the action of an external surface load (q_1, q_2, q_n), then the forces $T_1 = T_{x_1}$, $T_2 = T_{x_2}$ and S are determined from the following system of differential equations [2, 3]

$$\frac{\partial}{\partial x_1}(BT_1) + \frac{\partial}{\partial x_2}(AS) + ABq_1 = 0,$$
$$\frac{\partial}{\partial x_2}(AT_2) + \frac{\partial}{\partial x_1}(BS) + ABq_2 = 0, \tag{4}$$
$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = q_n.$$

The membrane condition imposes a constraint on the temperature field $T(x_1, x_2)$ wherein the temperature moments at all points of the middle surface equal zero. Hence, the deformations $\varepsilon_1 = \varepsilon_{x_1}$, $\varepsilon_2 = \varepsilon_{x_2}$, γ_{12} of the middle surface for shells of the type under consideration satisfy the following conditions [2, 3]

$$\frac{\partial}{\partial x_1}(B\varepsilon_2) = \frac{\partial}{\partial x_2}(A\gamma_{12}), \tag{5}$$

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$$\frac{\partial}{\partial x_2}(A\varepsilon_1) = \frac{\partial}{\partial x_1}(B\gamma_{12}), \qquad (5)$$

$$\frac{\partial^2}{\partial x_1 \partial x_2}\gamma_{12} = 0$$

in case the changes in curvature and torsion equal zero.

Moreover, they are related to the forces originating in the shell and the temperature Θ by Hooke's law

$$\varepsilon_1 = \frac{1}{Eh}(T_1 - \mu T_2) + \alpha\Theta,$$

$$\varepsilon_2 = \frac{1}{Eh}(T_2 - \mu T_1) + \alpha\theta, \quad \gamma_{12} = \frac{2(1+\mu)}{Eh} S, \qquad (6)$$

where E = const, μ = const, h = const, and α = const are Young's modulus, the Poisson ratio, the shell thickness, and the temperature coefficient of linear expansion, respectively. By virtue of (6) we have from the last equation in (5)

$$S(x_1, x_2) = S(x_1, 0) + S(0, x_2) - S(0, 0). \qquad (7)$$

This means that the shearing force in the shell is determined completely by its values at $x_1 = 0$ and $x_2 = 0$. By using (5) and (6) we convert (4) into the following form

$$\frac{\partial}{\partial x_1}[(2+\mu)T_1 + T_2 + Eh\alpha\Theta] + 2(1+\mu)Aq_1 = 0,$$

$$\frac{\partial}{\partial x_2}[(2+\mu)T_2 + T_1 + Eh\alpha\Theta] + 2(1+\mu)Bq_2 = 0, \qquad (8)$$

$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = q_n.$$

Integrating the first two equations in (8) we have

$$[(2+\mu)^2 - 1]T_1(x_1, x_2) = (2+\mu)^2 T_1(0, x_2) - T_1(x_1, 0) +$$
$$+ (2+\mu)[T_2(0, x_2) - T_2(x_1, 0)] + (2+\mu)(Eh\alpha\Theta)(0, x_2) - (Eh\alpha\Theta)(x_1, 0) -$$
$$-(1+\mu)Eh\alpha\Theta + 2(1+\mu)\int_0^{x_2} Bq_2(x_1, \eta)\,d\eta - 2(2+\mu)(1+\mu)\int_0^{x_1} Aq_1(\xi, x_2)\,d\xi,$$

$$[(2+\mu)^2 - 1]T_2(x_1, x_2) = (2+\mu)^2 T_2(x_1, 0) - T_2(0, x_2) + \qquad (9)$$
$$+ (2+\mu)[T_1(x_1, 0) - T_1(0, x_2)] + (2+\mu)(Eh\alpha\Theta)(x_1, 0) - (Eh\alpha\Theta)(0, x_2) -$$
$$-(1+\mu)Eh\alpha\Theta + 2(1+\mu)\int_0^{x_1} Aq_1(\xi, x_2)\,d\xi - 2(2+\mu)(1+\mu)\int_0^{x_2} Bq_2(x_1, \eta)\,d\eta,$$

where

$$T_1(x_1, 0) = T_1(0, 0) + \frac{1}{2+\mu}\{T_2(0, 0) - T_2(x_1, 0) + (Eh\alpha\Theta)(0, 0) - 2(1+\mu)\int_0^{x_1} Aq_1(\xi, 0)\,d\xi\} -$$
$$-\frac{1}{(2+\mu)^2}\{(Eh\alpha\Theta)(x_1, 0) + (1+\mu)(Eh\alpha\Theta)\}; \qquad (10)$$

$$T_2(0, x_2) = T_2(0, 0) + \frac{1}{2+\mu}\{T_1(0, 0) - T_1(0, x_2) + (Eh\alpha\Theta)(0, 0) -$$
$$-2(2+\mu)\int_0^{x_2} Bq_2(0, \eta)\,d\eta\} - \frac{1}{(2+\mu)^2}\{(Eh\alpha\Theta)(0, x_2) + (1+\mu)(Eh\alpha\Theta)\}.$$

Consequently, the third equation in (8) yields

$$\frac{\varphi''(x_1)}{1+[\varphi'(x_1)]^2}\{(2+\mu)^2 T_1(0,\ x_2) - T_1(x_1,\ 0) + (2+\mu)[T_2(0,\ x_2) -$$

$$- T_2(x_1,0)] - 2(2+\mu)(1+\mu)\int_0^{x_1} Aq_1(\xi,\ x_2)\,d\xi + 2(1+\mu)\int_0^{x_2} Bq_2(x_1,\ \eta)\,d\eta +$$

$$+ (2+\mu)(Eh\alpha\Theta)(0,\ x_2) - (Eh\alpha\Theta)(x_1,\ 0) - (1+\mu)\,Eh\alpha\Theta\} +$$

$$+ \frac{\psi''(x_2)}{1+[\psi'(x_2)]^2}\{(2+\mu)^2 T_2(x_1,\ 0) - T_2(0,\ x_2) + (2+\mu)[T_1(x_1,\ 0) - \qquad (11)$$

$$- T_1(0,\ x_2)] - 2(2+\mu)(1+\mu)\int_0^{x_2} Bq_2(x_1,\ \eta)\,d\eta + 2(1+\mu)\times$$

$$\times \int_0^{x_1} Aq_1(\xi,\ x_2)\,d\xi + (2+\mu)(Eh\alpha\Theta)(x_1,\ 0) - (Eh\alpha\Theta)(0,\ x_2) -$$

$$- (1+\mu)\,Eh\alpha\Theta\} = -q_n[(2+\mu)^2 - 1]\sqrt{1+[\varphi'(x_1)]^2+[\psi'(x_2)]^2}.$$

Formula (11) is fundamental in the solution of the problems formulated at the beginning of the paper. By using it equations can be deduced to find such functions $\varphi(x_1)$ and $\psi(x_2)$ for which a given surface load (q_1, q_2, q_n) and boundary conditions $T_1(0,\ x_2)$, $T_2(x_1,\ 0)$, $S(0,\ x_2)$, $S(x_1,\ 0)$ develop a flexure-free (i.e., membrane) deformation in a shell with the middle surface (1).

Let us examine a particular case of this problem when

$$q_1 = q_2 = 0,\ q_n = \text{const},\ \alpha = \text{const},\ \Theta = \text{const},$$
$$T_1(0,\ x_2) = T_1^0 = \text{const},\ T_2(x_1,\ 0) = T_2^0 = \text{const},\ S(0,\ x_2) = S(x_1,\ 0) = 0. \qquad (12)$$

In this case $S(x_1,\ x_2) = 0$, $T_1(x_1,\ x_2) = T_1{}^0$, $T_2(x_1,\ x_2) = T_2{}^0$. Consequently, the following equation results from (11)

$$T_1^0\frac{\varphi''(x_1)}{1+[\varphi'(x_1)]^2} + T_2^0\frac{\psi''(x_2)}{1+[\psi'(x_2)]^2} = -q_n\sqrt{1+[\varphi'(x_1)]^2+\psi'(x_2)]^2}. \qquad (13)$$

Since $A\approx 1$, $B\approx 1$, then within the limits of this accuracy we replace (13) by the following equation

$$T_1^0\frac{\varphi''(x_1)}{1+[\varphi'(x_1)]^2} + T_2^0\frac{\psi''(x_2)}{1+[\psi'(x_2)]^2} = -q_n\left\{1+\frac{1}{2}[\varphi'(x_1)]^2+\frac{1}{2}[\psi'(x_2)]^2\right\}, \qquad (14)$$

from which

$$T_1^0\frac{\varphi''(x_1)}{1+[\varphi'(x_1)]^2} + \frac{1}{2}q_n\{1+[\varphi'(x_1)]^2\} = \lambda,$$
$$T_2^0\frac{\psi''(x_2)}{1+[\psi'(x_2)]^2} + \frac{1}{2}q_n\{1+[\psi'(x_2)]^2\} = -\lambda, \qquad (15)$$

where λ is a constant that is still undetermined.

Equations (15) are nonlinear differential equations for φ and ψ:

$$T_1^0\varphi''(x_1) + \frac{1}{2}q_n\{1+[\varphi'(x_1)]^2\}^2 - \lambda\{1+[\varphi'(x_1)]^2\} = 0,$$
$$T_2^0\psi''(x_2) + \frac{1}{2}q_n\{1+[\psi'(x_2)]^2\}^2 + \lambda\{1+[\psi'(x_2)]^2\} = 0. \qquad (16)$$

Let us attach boundary conditions to (15)

$$\varphi(a)=\varphi(0)=0;\ \varphi\left(\frac{a}{2}\right)=\frac{H}{2};$$

$$\psi(b)=\psi(0)=0;\quad \psi\left(\frac{b}{2}\right)=\frac{H}{2};$$

$$x_3\left(\frac{a}{2},\frac{b}{2}\right)=H.$$

By using the substitutions

$$\varphi'(x_1)=u,\quad \psi'(x_2)=v \tag{17}$$

Eqs. (16) can be reduced in order. Integrating the equations that occur here, we obtain the following approximate formula for the desired mode of the membrane transfer surface:

$$x_3=\varphi(x_1)+\psi(x_2)=-\frac{2\lambda-q_n}{4T_1^0}x_1(a-x_1)+\frac{q_n+2\lambda}{4T_2^0}x_2(b-x_2),$$

where

$$\lambda=\frac{q_n[a^2T_2^0+b^2T_1^0]-16HT_1^0T_2^0}{2[a^2T_2^0-b^2T_1^0]}.$$

This formula has the same order of accuracy as the initial assumption (2) on the shallowness of the shell.

NOTATION

T_1, T_2, S are generalized forces acting at normal sections of the shell; ε_1, ε_2, γ_{12} are changes in linear elements of the middle surface and the angles between them; Θ is the change in temperature averaged over the shell thickness; E, μ are Young's modulus and the Poisson ratio of the shell; h is the thickness; α is the coefficient of temperature expansion; A, B, R_1, R_2 are coefficients of the first quadratic form, and the ratio of principal curvature of the shell middle surface; q_1, q_2, q_n are external surface load components in the direction of the coordinate lines and the normal; T is the temperature field.

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